

A Note on Return Regressions

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Abstract What is the relationship between firm values before investments, the investment costs firms pay, and the realized changes in firm values? We derive a formula that decomposes the marginal effect of initial firm values on realized returns into an *economic* effect and a *mechanical* effect, taking into account the endogeneity of investment. We show that regressing realized returns on initial firm values results in a biased estimate of the economic effect and that the bias direction and magnitude depends on relative investment size and realized returns. For data with positive returns, correcting the bias is straightforward. For data in which returns take both signs (e.g. acquirer returns in takeovers) stronger assumptions are necessary to interpret return regressions. We propose a local estimation procedure for circumventing this issue.

Key words: Investment, Return, Size Effects, Takeovers.

JEL Classification: G11, G14, G34

1 Introduction

Firms make investments, expecting improvement in firm values. Generally, firms making investments have different market values, incur different investment costs, leading to different realized returns. What is the relationship between

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firm values before investments, the investment costs, and the realized returns? The importance of understanding how firm values affect their investment decisions and associated returns has long been recognized in the corporate finance literature. However, no simple, theoretical derivation of the relationship among firm values, investment spending, and realized returns that can guide empirical works seems to be available. We attempt to fill this gap.

We frame the problem as follows. First, empirically, each investment episode contains three numbers: a firm value before the investment (V), investment cost (P), and a firm value after the investment ($\widehat{V}$). From $(V, P, \widehat{V})$, we can calculate a return on investment by $\pi \equiv \frac{\widehat{V}-V}{P}$ (changes in firm values per unit cost of investment) and a realized return by $R \equiv \frac{\widehat{V}-V}{V}$ (a rate of changes in firm values). The two definitions imply the identity $R = \pi \frac{P}{V}$. Second, theoretically, if we believe that a firm value V and an investment cost P matter *at the individual investment level*, we should anticipate some systematic relationship between π and (V, P) . This can be formalized by modelling π as a function of V and P . Then, we can ask the following question: *Is a realized return R informative about how V affects π ? More specifically, does regressing R on V reveal an economic impact of V on π ?*

The difficulty of answering this question stems from the fact that V affects $R = \pi \frac{P}{V}$ through three channels:

1. V affects R through its effect on π (*a firm-value effect*).
2. V affects R through the choice of P because:
 - (a) P affects π (*a matched investment-cost effect*).
 - (b) P multiplies $\frac{\pi}{V}$ (*a matched investment-scale effect*).
3. V divides $\pi P = \widehat{V} - V$ (*a firm-scale effect*).

First, the firm-value effect (1. above) is the extent to which an increase in V raises a return on a given investment, taking other factors fixed. Second, the two matched investment effects (2. above) come from firms' investment choice. For example, firms with larger V might optimally choose larger investments (i.e., P and V are positively associated across firms). Because changes in P may directly affect π (i.e., 2(a)) or scale up returns for a given level of π (i.e., 2(b)), the endogenous relationship between P and V creates an indirect chan-

nel through which changes in V affect R . While indirect, these effects admit economic interpretations.

In contrast, the firm-scale effect (3. above) is purely mechanical, following from *the way we define returns*. As a simple example, suppose that $\pi P = \widehat{V} - V$ is a positive random variable independent of V and independent across firms. For example, they may be purely driven by *luck*. Then, we should find that, $R = \frac{\pi P}{V}$ would be smaller on average for firms with larger V . Clearly, this relationship should *not* be interpreted as an *economic* impact of V . If we would like to learn *how V affects π* by regressing R on V , we need to isolate the firm-value effect from the other effects.

This paper is motivated by a large empirical literature on determinants of *acquirer returns in takeovers*, recently reviewed by Schneider and Spalt (2022). In a context of takeovers, V is a value of an acquiring firm and P is a price of a target firm. We present a general framework that nests a pure scaling model of Schneider and Spalt (2022) as a special case.

Our model has implications for a broader asset pricing literature that uses a measure of firm size as a control variable for cross-sectional variations in stock returns, as long as it is positively associated with market capitalization. Like Berk (1995), our model indicates a mechanical relationship between firm size and returns. The key difference is that while Berk (1995) focuses on expected returns, hence it is a story of risk, we study realized returns. Because our mechanism is driven by variations in returns for any reasons unrelated to initial firm values, the identified mechanical relationship holds regardless of investors' risk attitudes. Finally, while we present our model in a context of *heterogenous firms making financial investments*, the same logic applies to any individuals or organizations making costly investment, expecting improvement in their values, i.e., "returns". As such, we clarify the mechanism that has broad implications for empirical works that use return regressions.

The paper is organized as follows. Section 2 introduces a theoretical framework. Section 3 contains our main analysis. Section 4 studies a case with negative returns. Section 5 studies alternative regression specifications. Section 6 concludes the paper.

2 A theoretical framework

Each transaction (an investment episode) is identified by three numbers: an initial firm value V , its investment spending P , and the post-investment firm value $\widehat{V}$. A *return on investment* (henceforth ROI) is defined by changes in firm values per unit of investment cost:

$$\pi \equiv \frac{\widehat{V} - V}{P}. \quad (1)$$

Therefore, V and $\widehat{V}$ are connected through π by

$$\widehat{V} = V + \pi P. \quad (2)$$

By definition, ROI is positive if the firm value increases after the investment (i.e., $\widehat{V} > V$), and negative if $\widehat{V} < V$. We define a *realized return* by

$$R \equiv \frac{\widehat{V} - V}{V} = \pi \frac{P}{V}. \quad (3)$$

By assuming $P > 0$ and $V > 0$, (3) implies that R and π have the same sign.

So far, we only used two *definitions* to construct a new data (π, R) from a raw data $(V, P, \widehat{V})$. Variations in π constructed by (1) can be driven by many factors in actual data. Uncovering a systematic dependence of π on V is our goal. To achieve this goal, we need to assume some economic structure. We make two assumptions. First, we assume that some economic mechanisms exist through which both P and V may affect π in (2). For example, for a given investment opportunity, a firm can take various actions to increase its ROI, and the extent to which such actions are effective may depend on V . Without imposing a particular functional form, we write $\pi(V, P; \varepsilon)$, where a vector of random variables ε captures other factors that affect π *independent of* (V, P) . We call ε *ROI shocks*, or *shocks* for short.¹⁾ Second, we assume that firms with V *choose* P *to maximize* $\widehat{V}$ given in (2), knowing π and how it depends on P . The optimal

1) Return regressions often leave the overall variation in returns largely unexplained. For example, in a context of takeovers, Golubov et al. (2015) find that a regression using most of the commonly used variables explains only 6.4% of the variation in acquirer returns. They also show that adding firm-specific fixed effects doubles the model's explanatory power, but still much of the variation remains unexplained. Shocks ε capture residual variations in π unexplained by V , P , and other control variables.

choice by firms implies an *equilibrium relationship* between P and V .²⁾

If we are interested in return variations *attributable to initial firm values* V , it is best captured by $\pi_V \equiv \frac{\partial \pi(V, P; \varepsilon)}{\partial V}$ in this model. For example, π_V may be positive because of larger firms' advantage in negotiation or financing associated with the implementation of investments, or managerial skills to deal with agency problems. Accordingly, we ask the following questions: can regressing R on V reveal information about π_V ? How is the estimated regression coefficient biased relative to π_V ?

Suppose that firms observe ε at the time of investment. A firm with V solves

$$\max_P \{ \pi(V, P; \varepsilon) P + V \}.$$

Assuming $\pi(V, P; \varepsilon) > 0$ for some P , the first order condition is

$$\pi(V, P; \varepsilon) + P\pi_P = 0, \quad \text{where } \pi_P \equiv \frac{\partial \pi(V, P; \varepsilon)}{\partial P}. \quad (4)$$

The next assumption ensures that (4) uniquely defines the interior optimum $P^*(V; \varepsilon)$.

Assumption 1 $\pi_P < 0 < \pi$ and $\frac{\partial \pi_P}{\partial P} < -2\frac{\pi_P}{P}$ for any V, P, ε .

Assumption 1 states that investment is profitable, ROI is decreasing in investment cost P , and $\widehat{V} = \pi P + V$ is well-behaved as a function of P . From a theoretical point of view, it is a simple set of assumptions that connects data with a rational investment choice. From an empirical point of view, it can be checked with data π and P .

By substituting $P^*(V; \varepsilon)$, defined by (4), into a realized return (3),

$$R(V; \varepsilon) = \pi(V, P^*(V; \varepsilon); \varepsilon) \frac{P^*(V; \varepsilon)}{V}. \quad (5)$$

Regressing realized returns R on initial firm values V is expected to reveal an *economic mechanism* in (5). In the next section, we study the implication of (4)

2) In a more general model, actions of traders in a financial market determine V , taking into account the behavior of firms. In such an extended model, firms and traders jointly determine V and $\widehat{V}$. Note, however, that as long as firms take V as given when they make investments, the current analysis is valid, because it holds for any value of V , including those that would arise endogenously in the extended model.

and (5) for the interpretation of a regression coefficient obtained by regressing R on V .

3 Implications for regression analyses

In this section, we show that when realized returns are largely driven by forces that are independent of initial firm values and investment costs (in the language of our model, ROI shocks ε are dominant drivers of π), then one may obtain a statistically significant regression coefficient, but it does not reveal economic effects.

3.1 Interpreting a regression coefficient

We interpret a regression coefficient obtained by regressing R on V through the lens of our model. By taking a total derivative of (5) with respect to V ,

$$\frac{dR}{dV} = \underbrace{\pi_V \frac{P^*}{V}}_1 + \underbrace{\frac{P_V^*}{V} (P^* \pi_P + \pi)}_{2(a)(b)} - \underbrace{\frac{R}{V}}_3, \quad (6)$$

where $\pi_V \equiv \frac{\partial \pi(V, P; \varepsilon)}{\partial V}$ and $P_V^* \equiv \frac{\partial P^*(V; \varepsilon)}{\partial V}$. The first term $\pi_V \frac{P^*}{V}$ is the firm-value effect. The second term is the matched investment effects. The third term $-\frac{R}{V}$ is the firm-scale effect. We combine (6) with the optimality condition (4) for the investment choice.

Lemma 1 *If the optimality condition (4) characterizes $P^*(V; \varepsilon)$, then (6) becomes*

$$\frac{dR}{dV} = \pi_V \frac{P^*}{V} - \frac{R}{V}. \quad (7)$$

Proof. The optimality condition (4) implies that the second term in (6) is zero. ■

Lemma 1 shows that in the data in which P^* and V are connected by the optimality condition (4), the total marginal impact of V on R is the sum of two terms: (i) π_V multiplied by $\frac{P^*}{V}$, and (ii) $-\frac{R}{V}$. In a context of making an inference about π_V , we call $\frac{P^*}{V}$ the *multiplicative bias* and $-\frac{R}{V}$ the *additive bias*.

First, the multiplicative bias $\frac{P^*}{V}$ is the *relative investment size* and a direction of this bias depends on the sign of π_V . We argue that $\frac{P^*}{V}$ is smaller than one in

most applications, because V is market capitalization while P^* is investment expenditure.³⁾ Then, $\pi_V \frac{P^*}{V}$ underestimates positive π_V and overestimates negative π_V .

Second, because $R = \pi \frac{P^*}{V}$ must be positive for (4) to be a valid optimality condition, the additive bias $-\frac{R}{V}$ creates a downward bias. The magnitude of the additive bias increases in R and decreases in V , suggesting more significant downward bias for data in which V is small relative to R . **Table 1** summarizes these implications.

Table 1. Comparison of $\frac{dR}{dV} = \pi_V \frac{P^*}{V} - \frac{R}{V}$ with π_V

Direction of multiplicative bias $\frac{P^*}{V}$		Total direction of bias
$\frac{P^*}{V} < 1$	underestimate $\pi_V > 0$	$\frac{dR}{dV} < \pi_V$
	overestimate $\pi_V < 0$	$\frac{dR}{dV} \lessgtr? \pi_V$
$\frac{P^*}{V} > 1$	overestimate $\pi_V > 0$	$\frac{dR}{dV} \lessgtr? \pi_V$
	underestimate $\pi_V < 0$	$\frac{dR}{dV} < \pi_V$

If $\frac{P^*}{V} < 1$ and $\frac{P^*}{V}$ decreases in V , then the multiplicative bias becomes more significant for data in which V is larger on average. The condition that $\frac{P^*}{V} < 1$ and decreasing in V is likely to hold in many applications, because $\frac{P^*}{V}$ is a ratio of particular investment expenditures to overall values of those making investment. On the other hand, for a given $R > 0$, the additive bias $-\frac{R}{V}$ is negative and its magnitude decreases in V . This indicates the following trade-off: for data in which V is larger on average, the magnitude of the additive bias $-\frac{R}{V}$ would be smaller, but smaller $\frac{P^*}{V}$ makes $\frac{dR}{dV}$ further away from π_V .

Remark 1. In studying cross-section of stock returns, Fama and French (2008) document that the size anomaly is concentrated in “microcaps” (firms with very small V). More generally, a number of recent studies find that the size premium is decreasing in firm size.⁴⁾ Note that (7) applied to abnormal returns

3) Even for applications where individuals make investments (e.g. education), V is a stock variable (e.g. human capital) while P^* is a flow variable (expenditure on education). So $\frac{P^*}{V}$ is likely to be small.

4) See Alquist et al. (2018) (Exhibit 18 and 19 on page 49), Asness et al. (2018) (Figure 6 on page 496), and Jensen et al. (2023) (Figure 12 on page 47).

predicts that $-\frac{R}{V}$ is large negative for small V . Because (7) is a *marginal* effect of V on R , if it takes a large negative value, it predicts that positive abnormal returns on small firms should quickly disappear as V increases, consistent with their concentration among very small firms.

3.2 Bias correction in return regressions

Lemma 1 suggests a straightforward way to make an inference about π_V . From (7),

$$\frac{V}{P^*} \left(\frac{dR}{dV} \right) + \frac{R}{P^*} = \pi_V. \quad (8)$$

This suggests the following bias correction procedure. First, for each quantile of data sorted by V , obtain an estimate of $\frac{dR}{dV}$ by regressing R on V . Second, adjust the coefficient by (8) using $(\frac{V}{P^*}, \frac{R}{P^*})$ for the corresponding quantile of data. This procedure allows for the estimation of π_V for each quantile of V .

4 Negative realized returns

A drawback of the model studied in the previous section is that it is not consistent with *negative* realized returns. For example, acquirer returns in takeovers are often negative. To rationalize negative returns within the current framework, we must assume that at least some element of ε must be unobservable to firms at the time of investment, and also that some realizations of those shocks make returns negative. Under this alternative assumption, a firm with V solves

$$\max_P \{ \bar{\pi}(V, P) P + V \}, \quad \text{where } \bar{\pi}(V, P) \equiv E[\pi(V, P; \varepsilon)] > 0.$$

Importantly, assuming $\bar{\pi}(V, P) > 0$ does not exclude $\pi(V, P; \varepsilon) < 0$ for some realizations of ε . The first order condition is

$$\bar{\pi}(V, P) + P \bar{\pi}_P = 0, \quad \text{where } \bar{\pi}_P \equiv \frac{\partial \bar{\pi}(V, P)}{\partial P}. \quad (9)$$

The next assumption ensures the interior optimum $\bar{P}^*(V)$ defined by (9).

Assumption 2 When ε is unobservable, $\bar{\pi}_P < 0 < \bar{\pi}$ and $\frac{\partial \bar{\pi}_P}{\partial P} < -2 \frac{\bar{\pi}_P}{P}$ for any V, P .

With $\bar{P}^*(V)$ implicitly defined by (9), a realized return is

$$R(V; \varepsilon) = \pi \left(V, \bar{P}^*(V); \varepsilon \right) \frac{\bar{P}^*(V)}{V}. \quad (10)$$

4.1 Interpreting a regression coefficient with negative returns

Realized returns (10) depend on π , but **Assumption 2** does not determine the sign of π and π_P . Therefore, we need more assumptions to study the implication of (9) and (10) for the regression analysis.

By taking a total derivative of the realized return (10) with respect to V ,

$$\frac{dR}{dV} = \underbrace{\pi_V \frac{\bar{P}^*}{V}}_1 + \underbrace{\frac{\bar{P}_V^*}{V} \left(\bar{P}^* \pi_P + \pi \right)}_{2(a)(b)} \underbrace{- \frac{R}{V}}_3. \quad (11)$$

By using the optimality condition (9), we obtain the following result.

Lemma 2 *If the optimality condition (9) characterizes $\bar{P}^*(V)$, then (11) becomes*

$$\frac{dR}{dV} = \begin{cases} \frac{\bar{P}^*}{V} \pi_V - \frac{R}{V} \left\{ 1 + \bar{P}_V^* \frac{V}{\bar{P}^*} \left(\frac{\pi_P/\pi}{\bar{\pi}_P/\bar{\pi}} - 1 \right) \right\} & \text{for } R \neq 0, \\ \frac{\bar{P}^*}{V} \left(\pi_V + \bar{P}_V^* \pi_P \right) & \text{for } R = 0. \end{cases} \quad (12)$$

Proof. For $R = 0$, substitute $R = \pi = 0$ in (11). For $R \neq 0$, use $\bar{P}^* = -\frac{\bar{\pi}}{\bar{\pi}_P}$ to write $\bar{P}^* \pi_P + \pi$ in (11) as $-\pi \left(\frac{\pi_P/\pi}{\bar{\pi}_P/\bar{\pi}} - 1 \right)$. Use $\frac{\pi}{V} = \frac{R}{\bar{P}^*}$ to obtain the result. ■

In (12), the additive bias for $R \neq 0$ can be written as

$$-\frac{R}{V} \left\{ 1 + \eta_V^{\bar{P}^*} \left(\frac{\eta_P^\pi}{\eta_P^{\bar{\pi}}} - 1 \right) \right\}, \quad (13)$$

where $\eta_V^{\bar{P}^*} \equiv \bar{P}_V^* \frac{V}{\bar{P}^*}$ is the elasticity of the optimal investment $\bar{P}^*(V)$, $\eta_P^\pi \equiv \pi_P \frac{P}{\pi}$ is the elasticity of π with respect to P , and $\eta_P^{\bar{\pi}} \equiv \bar{\pi}_P \frac{P}{\bar{\pi}}$ is the elasticity of $\bar{\pi}$ with respect to P . If $\eta_P^\pi = \eta_P^{\bar{\pi}}$, then (13) collapses to $-\frac{R}{V}$. Otherwise, we must sign (13) to determine the direction of the additive bias. By **Assumption 2**, $\eta_P^{\bar{\pi}} < 0$. The sign of $\eta_V^{\bar{P}^*}$ can be assessed empirically. However, the term $\left(\frac{\eta_P^\pi}{\eta_P^{\bar{\pi}}} - 1 \right)$ is

difficult to sign empirically and theoretically.⁵⁾

The bias in the presence of negative returns is harder to sign, hence more difficult to correct. In the next subsection we propose a way to make an inference about π_V when the additive bias in (13) is large and difficult to sign.

4.2 Bias correction

In general, the direction and the magnitude of the additive bias (13) are difficult to know. To minimize the effect of the additive bias, we propose a *local regression with observations* $R \approx 0$. Specifically, (12) for $R = 0$ implies

$$\frac{V}{\bar{P}^*} \left(\frac{dR}{dV} \right) \Big|_{R=0} = \left(\pi_V + \bar{P}_V^* \pi_P \right) \Big|_{\pi \approx 0}. \quad (14)$$

This suggests a local inference using observations with $R \approx 0$. We still need to know $\bar{P}_V^* \pi_P \Big|_{\pi \approx 0}$ to estimate $\pi_V \Big|_{\pi \approx 0}$. However, if we can sign $\bar{P}_V^* \pi_P$ around $\pi \approx 0$, (14) could be used to sign π_V around $\pi \approx 0$. More precisely,

$$\begin{aligned} \frac{V}{\bar{P}^*} \left(\frac{dR}{dV} \right) \Big|_{R \approx 0} > 0 &\Rightarrow \pi_V \Big|_{\pi \approx 0} > -\bar{P}_V^* \pi_P \Big|_{\pi \approx 0}, \\ \frac{V}{\bar{P}^*} \left(\frac{dR}{dV} \right) \Big|_{R \approx 0} < 0 &\Rightarrow \pi_V \Big|_{\pi \approx 0} < -\bar{P}_V^* \pi_P \Big|_{\pi \approx 0}. \end{aligned}$$

For example, if one finds $\frac{V}{\bar{P}^*} \left(\frac{dR}{dV} \right) \Big|_{R \approx 0} > 0$, $\bar{P}_V^* > 0$, and $\pi_P \Big|_{\pi \approx 0} < 0$ in the data, then $\pi_V \Big|_{\pi \approx 0}$ has a *positive lower bound* $-\bar{P}_V^* \pi_P \Big|_{\pi \approx 0}$. In the next subsection, we use this result to interpret some results presented by Schneider and Spalt (2022).

4.3 Discussion of Schneider and Spalt (2022)

To interpret acquirer returns in takeovers, where V is a bidder firm value, P is a price of a target firm, Schneider and Spalt (2022) use a “pure scaling model” with $\pi_P = \pi_V = 0$. Their main message is that the term $-\frac{R}{V}$ (i.e., scaling effect) mostly drives acquirer size effects. They report that they cannot reject $\pi_P = 0$

5) By the implicit function theorem applied to (9), $\bar{P}_V^* = \frac{1}{V} \frac{\eta_V^{\bar{\pi}} - \eta_V^{\bar{\pi}_P}}{1 - \eta_P^{\bar{\pi}} - \eta_P^{\bar{\pi}_P}}$. Thus, the sign of $\bar{P}_V^*$ (i.e., whether $\bar{P}^*(V)$ increases or decreases in V) is informative about the elasticities of $\bar{\pi}$ and $\bar{\pi}_P$ with respect to V and P . However, it does not sign $\left(\frac{\eta_P^{\bar{\pi}}}{\eta_P^{\bar{\pi}_P}} - 1 \right)$.

in their data. However, while statistically insignificant, their Table 6 exhibits $\pi_P < 0$ (especially for Non-Public firms, for which $\pi > 0$ on average).

For the local estimation based on (14), Figure 2 in Schneider and Spalt (2022) is suggestive. They estimated $\frac{dR}{dV}$ for each decile of R . If $\frac{dR}{dV} = -\frac{R}{V}$, as in their “pure scaling model” with $\pi_P = \pi_V = 0$, then the estimated coefficient should be *close to zero for the decile containing $R = 0$* . We denote the estimated coefficient for such decile by $\frac{dR}{dV}|_{R \approx 0}$. The authors in fact find $\frac{dR}{dV}|_{R \approx 0} < 0$ for their subsample with non-public targets, and $\frac{dR}{dV}|_{R \approx 0} > 0$ for the remaining subsample with public targets. Both findings are statistically significant at the 5% confidence level.⁶⁾

To further interpret these local estimation results, note that their size sorts exhibit $\bar{P}_V^* > 0$, regardless of subsamples (their Table 3 and Table 5). While they do not report the value of $\pi_P|_{\pi \approx 0}$ for the corresponding decile, if $\pi_P|_{\pi \approx 0} < 0$ holds for their subsample with public targets, then (14) indicates $\pi_V|_{\pi \approx 0} > -\bar{P}_V^* \pi_P|_{\pi \approx 0} > 0$. That is, conditional on $\pi_P|_{\pi \approx 0} < 0$, firms with larger V have higher ROI (locally around $\pi \approx 0$) when taking over public targets. This additional implication does not dispute their main message (that the scaling effect is the main driver of the acquirer size effect). We only add to their work by pointing out that the location of the flipping point (the decile where estimated effects change signs) can be linked to π_V and π_P through (14), a fact ignored in their pure scaling model.

5 Alternative regression specifications

One may wonder if alternative regression specifications may lead to reduced biases. We study two specifications: regressing R on $\frac{P}{V}$, and regressing π on V .

Regressing R on $\frac{P}{V}$. This regression is motivated by Jansen et al. (2013). They also study acquirer returns in takeovers, using a sample in which a significant fraction of realized returns are negative. Jansen et al. (2013) note that preceding studies which *regress R on relative target size $\frac{P}{V}$* found a positive co-

6) In their Table 7, $R = 0$ occurs between 40th and 50th percentiles for the non-public target sample, and between 50th and 60th percentiles for the public target sample. See the corresponding part of blue solid lines (bidder size coefficient estimates) in their Figure 2, panel (b) for the non-public target sample and panel (c) for the public target sample.

efficient estimate (e.g., Asquith et al. (1983)) as well as a negative estimate (e.g., Fuller et al. (2002)). Moreover, they show that when they split their sample by the sign of R , the coefficient estimate in each sample has the corresponding sign of R (p.535, Figure 1).⁷⁾ The next result sheds light on their findings.

Recall that $\bar{P}^*$ is the interior optimum, implicitly defined by (9). Taking a total derivative of the realized return (10) with respect to $\frac{\bar{P}^*}{V}$ yields the next result.

Lemma 3 $\frac{dR}{d\left(\frac{\bar{P}^*}{V}\right)} = \frac{V}{\bar{P}^*} \left(R + \pi_P \bar{P}^* - \pi_V V \right).$

Proof. Let $k \equiv \frac{\bar{P}^*}{V}$. Given $\pi = \pi(V, \bar{P}^*) = \pi\left(\frac{\bar{P}^*}{k}, V\right)$ and $R = \pi k$, $\frac{dR}{dk} = \pi + \pi_V \left(-\frac{\bar{P}^*}{k^2}\right) + \pi_P V$. Use $\pi_V \left(-\frac{\bar{P}^*}{k^2}\right) = -\frac{V}{\bar{P}^*} \pi_V V$ and $\pi = \frac{V}{\bar{P}^*} R$ to obtain the result. ■

Lemma 3 shows that the regression coefficient and R have the same sign if $\pi_P \bar{P}^* - \pi_V V \approx 0$. This would be the case if variations in π are largely driven by ε , not by (P, V) .

Regressing π on V . For this regression specification, we focus on a case with positive returns. Recall that P^* is the interior optimum, implicitly defined by (4).

Lemma 4 Let $\eta_V^{P^*} \equiv P_V^* \frac{V}{P^*}$. If the optimality condition (4) characterizes $P^*(V; \varepsilon)$, then the total marginal effect of V on π is

$$\frac{d\pi}{dV} = \pi_V - \eta_V^{P^*} \frac{R}{P^*}. \quad (15)$$

The bias $-\eta_V^{P^*} \frac{R}{P^*}$ in (15) has the same sign as $-\frac{R}{V}$ in (7), and is smaller in magnitude, if and only if

$$0 < \eta_V^{P^*} < \frac{P^*}{V}. \quad (16)$$

Proof. To obtain (15), substitute $\pi_P = -\frac{\pi}{P^*}$ into $\frac{d\pi}{dV} = \pi_V + \pi_P P_V^*$ and use $\eta_V^{P^*} \equiv P_V^* \frac{V}{P^*}$ and $\frac{\pi}{V} = \frac{R}{P^*}$. The two inequalities in (16) follow by comparing

7) Similar results are reported in Alexandridis et al. (2017) and Schneider and Spalt (2022).

$$-\frac{\eta_V^{P^*}}{P^*}R \text{ and } -\frac{R}{V}. \quad \blacksquare$$

In (15), the sign of the bias $-\eta_V^{P^*} \frac{R}{P^*}$ depends on $\eta_V^{P^*}$. Hence, this regression specification adds an additional layer of uncertainty. If one is confident that the condition (16) holds, then for the purpose of learning π_V , regressing π on V leads to smaller bias than regressing R on V . We note, however, that (16) can be written as $0 < P_V^* < \left(\frac{P^*}{V}\right)^2$. For applications where $\frac{P^*}{V}$ is small, this condition may be violated.

6 Conclusion

We presented a simple analytical framework to guide empirical work that aims to measure the economic impact of firm values on returns on investment. Our model identifies a mechanical relationship between initial firm values and realized returns, driven by variations in returns on investment for reasons unrelated to initial firm values. The model has implications for asset pricing studies using firm size (market capitalization) as a control variable in return regressions. We also proposed a way to isolate (or circumvent) the potential bias in the interpretation of the statistical relationship between initial firm values and realized returns. We made our model as simple as possible to expose its basic logic. However, any firm- or transaction-level characteristics that are correlated with initial firm values would inherit the biases identified here. We expect that the basic logic persists through time, across industries, and regardless of institutional details such as regulations and transaction methods. Finally, measuring the magnitude of the bias for various applications is an important task. A recent empirical work by Schneider and Spalt (2022) in a context of takeovers suggests that such bias may be quite large.

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